Experimental Stability Studies in Wakes of Two-Dimensional Slender Bodies at Hypersonic Speeds

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Experimental stability studies were conducted in the transition region from laminar to turbulent flow in wakes of slender wedges and a flat plate at Mach number 6. As in low-speed flat plate wakes, transition from laminar to turbulent flow may be divided into a linear and a nonlinear instability region. Inviscid linear stability theory predicts well the growth of fluctuations and amplitude distribution in the linear region. In the nonlinear region similarities with low-speed wakes exist. Characteristic persisting peaks in the power spectra are observed. Based on these peak frequencies a nearly universal Strouhal number of $fb_0/u_\infty \doteq 0.3$ was found for both incompressible and hypersonic wake flows. A theoretical approach to predict the development of mean flow and flow fluctuations in the nonlinear region as employed by Ko, Kubota, and Lees in slender body low-speed wakes appears equally applicable for hypersonic wakes.

Nomenclature

b	= wake width
b_0	= wake width at neck based on velocity profile
c	= phase velocity ($= c_R + ic_I$)
c_g	= group velocity
e'	= hot-wire fluctuation voltage
Δe	= hot-wire sensitivity coefficient
f	= frequency
H	= wedge base height
i	= hot-wire current
L	= flat plate length
m	= mass flow ($= \rho u$)
M	= Mach number
R_w	= hot-wire resistance
Re	= Reynolds Number
S	= Strouhal number ($= fb/u_e$)
T_0	= total temperature
α	= wave number
δ	= wedge angle
ρ	= density
θ	= momentum thickness

Superscripts

$(\ )'$	= fluctuation quantity
$(\tilde{\ })'$	= rms of fluctuation quantity
$(\bar{\ })$	= mean flow quantity

Subscripts

$(\ )_e$	= wake edge quantity
$(\ )_f$	= quantity at frequency f
$(\ )_r$	= reference quantity
$(\ )_\infty$	= freestream quantity

Introduction

A GREAT number of measurements of transition from laminar to turbulent flow in hypersonic wakes of blunt and slender bodies, both two-dimensional and axisymmetric have been performed by many investigators (see Summary Paper, Ref. 1). These transition data were correlated in several different ways in order to find parameters governing hypersonic wake transition and/or to predict it.[1] The results of these correlations are such that one may very roughly predict transition in most cases, but the understanding of the phenomenon of transition gained from transition location measurements and correlations alone is limited.

The location of transition in two-dimensional wakes was obtained by hot-wire fluctuation and mean flow measuring techniques. In most axisymmetric wake measurements, transition was found by interpreting wake photographs (schlieren, shadowgraph, racetrack). According to Pallone et al.,[2] the location of transition was defined as the "occurrence of distinct and continuous waviness." Not only distinct and continuous waviness, but definitely periodic waviness has been observed by many investigators.[1,3]

At hypersonic speeds the shear layers stemming from the body boundary layers are relatively stable.[4,5] Therefore, vortex shedding as has been observed behind blunt bodies at low-speeds appears unlikely. However, compressible wake flows are dynamically unstable because there exists an extremum in the density-vorticity-product (Lees and Lin[6]). On these grounds Lees and Gold[5] suggested that the hypersonic wake transition mechanism is qualitatively similar to transition behind thin slender bodies at low speeds.

Low-speed wake transition behind a flat plate was investigated by Taneda[7] in a water tow tank and by Sato and Kuriki,[8] using hot-wire technique. Taneda shows pictures of sinusoidal oscillations of laminar wakes behind a flat plate at low speeds. Farther downstream these sinusoidal oscillations develop into a double row vortex street ($Re_L > 10^3$). For higher Reynolds numbers the vortex street is soon deformed and destroyed.

More detailed but similar results were obtained by Sato and Kuriki, who investigated the wake of a flat plate in a wind tunnel. On the basis of their hot-wire measurements, they divided transition in a flat plate wake into three subregions: 1) the linear instability region in which the mean flow grows according to steady laminar boundary layer theory (Goldstein solution) and the fluctuations grow as predicted by two-dimensional inviscid stability theory; 2) the nonlinear region in which an antisymmetrical double row vortex street develops, i.e., in terms of the frequency spectrum, besides the fundamental frequency of oscillation, the first higher harmonic appears on and near the wake axis; initially in this nonlinear region the mean wake width grows much faster than a lami-

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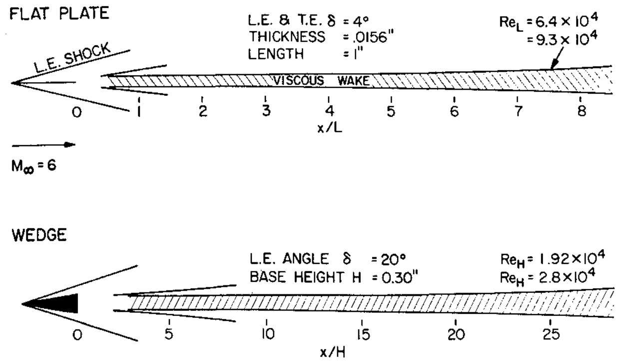

Fig. 1 Wake flow configurations investigated in detail.

nar steady flow; 3) the three-dimensional region characterized by three-dimensional distortions of the double row vortex street before the wake becomes "fully turbulent."

Assuming similar stages of transition occur in hypersonic wakes, the first step is to study linear stability of hypersonic wakes. This problem was studied theoretically by Lees and Gold.[5] Since wake profiles possess inflexion points they are dynamically unstable (Lees and Lin).[6] The main problem, therefore, is to find the modes of instability, the range of unstable frequencies (wave numbers), and the amplification rates of small perturbations. Experimental measurements of the amplification rates in the linear instability region at supersonic and hypersonic speeds were performed by Kendall[9] in the wake of a flat plate at $M_\infty = 3.7$, using artificial disturbances, by Behrens[10] in the far outer wake of circular cylinders and by Lewis and Behrens[11] in the wake of a wedge at $M_\infty = 4$, using natural disturbances. Calculating the rate of growth of fluctuations for Gaussian mean wake profiles, good agreement between linear stability theory and experiments was obtained by Kendall for a wake of a flat plate, where the mean wake profiles are very close to Gaussian. Agreement between theory and experiment was not as good in the other two cases. However, it has been shown[12] that good agreement between theory and experiment is again obtained for these cases when the measured mean wake profiles are used in the theoretical stability calculations.

Schlieren spark pictures of a wake behind a flat plate at $M_\infty = 2.4$ (Kendall[13]), artificially excited at one frequency, showed the appearance of a nonlinear region (antisymmetrical vortex street). Also, mean flow and hot-wire fluctuation measurements indicated the existence of a nonlinear instability region in the far outer wake behind circular cylinders (Behrens[10]) which exhibited qualitatively similar features as found by Sato and Kuriki[8] in the flat-plate wake. These results indicate that there is a qualitative similarity between the transition processes in high- and low-speed wakes. However, in comparison to investigations in low-speed wakes, the information on the transition mechanism in hypersonic flow is limited. Therefore, this problem has been investigated in the wakes of slender wedges and flat plates at Mach 6.

In these wakes the location of "transition," which is now being recognized as the onset of non-linearity, was determined earlier by means of hot-wire fluctuation measurements by Demetriades and Behrens[14] and also by Batt and Kubota[15] by means of mean flow measurements. It was found that at the onset of nonlinearity the hot-wire fluctuation signal on the wake axis which is nonmeasurably small in the linear region, increases. A short distance downstream, the mean wake width begins to widen rapidly and the wake velocity defect and temperature excess decrease sharply.

In the present experimental investigation the stability behavior of natural flow fluctuations in both the linear and nonlinear instability regions have been studied. Hot-wire fluctuation measurements were performed in this investigation to find the growth and decay of flow fluctuations and the development of frequency spectra. Mean flow measurements by Batt and Kubota[15] were used to reduce the data for comparison with linear theory. The experimental technique and data reduction procedure are explained in the next section. An account of the experimental results is then given which is followed by a discussion. The linear instability region is first discussed including a comparison with linear stability theory and then the results found in the nonlinear instability region are discussed and compared with observations found in low-speed wakes in the nonlinear region. In this section it is also shown that a similar theoretical model as used by Ko, Kubota, and Lees[16] in low-speed wakes should be applicable to predict the mean flow behavior and the development of the fluctuations of the most unstable frequency component as the flow moves from the linear into the nonlinear instability regime.

Experimental Technique and Data Reduction

Wind Tunnel and Models

The experiments were performed in the GALCIT Hypersonic wind tunnel Leg I, which is a closed-return, continuous flow wind tunnel with a 5-× 5-in.^{2} cross section and a nominal Mach number of 6. The Reynolds number ranges from 30,000/in. to 230,000/in. for a stagnation temperature of 275°F.

The models, spanning the test-section, were 10°-half angle wedges of base heights $H = 0.15$ in. and 0.30 in. and a one-inch flat plate of 0.015-in. thickness, with sharp leading and trailing edges. The models most frequently used in these experiments and the flow conditions are shown in Fig. 1.

Hot-Wire Fluctuation Measurements

A constant current hot-wire anemometer system was used to measure the growth and decay of the wake flow fluctuations in the flow direction, fluctuation distributions across the wake and frequency spectra. The hot wire was a 10^{-4}-in. platinum-10% rhodium hot-wire with an aspect ratio l/d of about 100. The time constant of the hot-wire thermal lag typically varied from 0.24 msec at the edge of the wake to 0.3 msec at the wake centerline. The electronic equipment consisted of a constant current hot-wire set with a frequency range between 1 Hz and 320 kHz (Shapiro-Edwards), wave analyzers, oscilloscope and X-Y plotters.

The time lag of the hot-wire response was compensated for at the edge of the wake using the square wave method (Kovasznay[17]). This time constant setting was sufficient since all fluctuation measurements were made in a frequency range of $f = 5$ to 200 kHz. It has been shown[18] that at large enough frequencies it is not necessary to know the hot-wire time constant exactly at each station for proper data reduction. The voltage fluctuation output of the compensating amplifier is given by $\tilde{e}_f{}'_{,\text{comp}} = \tilde{e}_f{}'_{,\text{ideal}}\ [1 + (2\pi f M_A)^2]^{1/2}/[1 + (2\pi f M_t)^2]^{1/2}$, where M_A is the time constant setting of the compensating amplifier and M_t the hot-wire time constant. For frequencies $f \geq 5$ kHz and $M_t \simeq 0.25$ msec, $\tilde{e}_f{}'_{,\text{comp}} \simeq \tilde{e}_f{}'_{,\text{ideal}}\ M_A/M_t$. Thus, since also the amplifier has a transfer function nearly independent of frequency up to 200 kHz (the gain is decreased by 0.9 dB at 200 kHz) and the floor-to-ceiling ratio of the compensating amplifier is 500, the measured energy spectra of fluctuations in the range of 5 to 200 kHz are without distortion. Measurements were made at one current corresponding to an overheat of $(R_w - R_{aw})/R_{aw} = 0.20$ at the edge of the wake.

Hot-Wire Sensitivity

In supersonic flow, the unsteady hot-wire response is sensitive to total temperature and mass-flow fluctuations (Kovasznay,[17] Morkovin[19]), i.e., $e' = \Delta e_T T_0'/\bar{T}_0 - \Delta e_m m'/\bar{m}$, where Δe_T and Δe_m are the hot-wire total temperature and mass flow sensitivity coefficients, respectively. At large overheats these hot-wire sensitivity coefficients are of the same order of magnitude. Furthermore, in adiabatic wakes (Demetriades[20]) as well as adiabatic boundary layers (Kistler[21]) the total temperature fluctuations are small compared to the mass flow fluctuations. Therefore, for large overheats, the total temperature fluctuations may be neglected and the hot-wire response in supersonic flow is

$$\tilde{e}' \cong \Delta e_m \tilde{m}'/\bar{m} \qquad (1)$$

Data Reduction

The growth rates of fluctuations in the linear region are obtained as follows. Any fluctuation quantity, Q_n' grows spatially as $d \ln\tilde{Q}_n'/dx = \alpha c_I/c_g$ (see for example Lees and Gold[5]). Letting $Q_n' = m'/\Delta m_{\text{¢}}$, the properly normalized mass-flow fluctuation [$= \bar{m}/\Delta m_{\text{¢}}(\rho'/\bar{\rho} + u'/\bar{u})$ for small perturbations], the spatial rate of amplification becomes

$$\alpha c_I/c_g = d \ln(\tilde{m}'/\Delta m_{\text{¢}})/dx \qquad (2)$$

where $\Delta m_{\text{¢}} = (\rho u)_e - (\rho u)_{\text{¢}}$. The hot-wire anemometer measures the rms fluctuation voltage e_f', given by Eq. (1). Using these formulas, the rate of amplification may be calculated from the following equation

$$\alpha c_I/c_g = (1/\tilde{e}_f')d\tilde{e}_f'/dx - (1/\Delta e_m)d\Delta e_m/dx + (1/\tilde{m})d\tilde{m}/dx - (1/\Delta m_{\text{¢}})d\Delta m_{\text{¢}}/dx \qquad (3)$$

It is clear from this relation that the axial mean flow derivatives have to be taken into account in the data reduction of the local amplification rates of fluctuations.

Experimental Results

On the basis of the mean flow and fluctuation measurements a division may be made between the linear and nonlinear instability regions. In the linear instability region the mean wake flow is predicted by the steady, two-dimensional boundary-layer theory (Batt and Kubota[15]). But at the onset of nonlinearity there is a sharp increase in wake width. This behavior of the wake behind a flat plate has been found by both mean flow measurements and hot-wire traces across the wake. The wake growth is shown in Fig. 2 for several Reynolds numbers. The wake flow at $Re_L = 4.6 \times 10^4$ is a laminar steady flow up to $x/L = 11$. At $Re_L = 6.4 \times 10^4$ nonlinear effects set in at $x/L \cong 8$ and at $Re_L = 9.3 \times 10^4$ at $x/L \cong 4.5$.

For the last case, typical mean square hot-wire voltage fluctuation traces across the wake are shown in Fig. 3. The fluctuations grow in the linear region (ahead of $x/L \cong 4.5$), but no fluctuation signal is measured on the wake axis. As the wake becomes nonlinear the signal on the axis starts to grow rapidly. In order to investigate the fluctuations which cause this breakdown of the steady laminar flowfield, frequency distributions of fluctuations were taken at points in the wakes

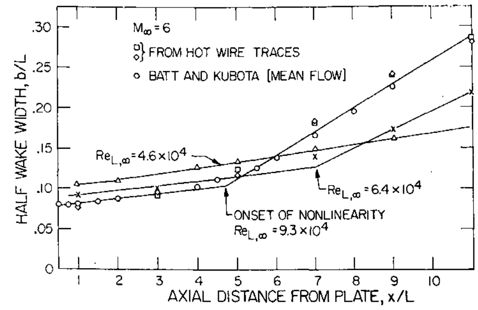

Fig. 2 Wake widths of flat plate wakes as function of Reynolds number.

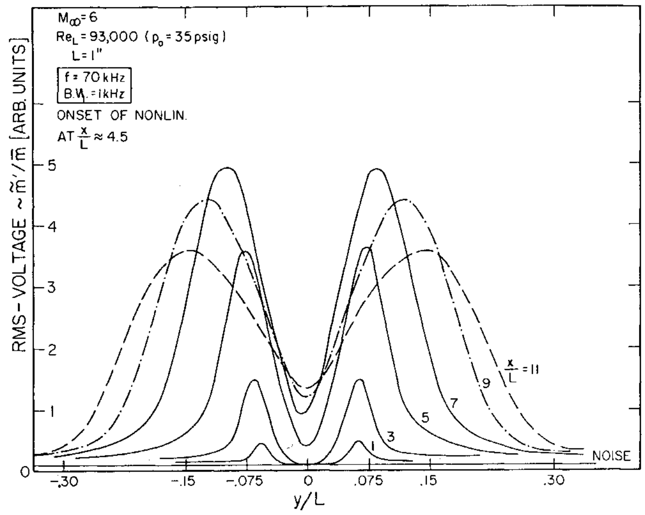

Fig. 3 Typical hot-wire fluctuation traces across a flat-plate wake (f = 70 kHz).

where the fluctuation signal is a maximum and on the wake centerline.

Frequency distributions were taken in the flat plate wake, for which the hot-wire traces were shown in Fig. 3. Near the flat plate ($x/L = 1$), the fluctuation intensity monotonically decays with increasing frequency (Fig. 4). As the flow moves downstream, a peak in the spectrum develops at 70 kHz. Even after the onset of nonlinearity, at first, the oscillations become even more tuned about 70 kHz (see Fig. 4, $x/L = 7$). But further downstream, the peak in the spectrum decreases rapidly (Fig. 5). Note that in the nonlinear region on the wake axis a relative maximum occurs at twice the most unstable frequency (Fig. 5, $x/L = 9$, centerline). The most unstable frequency will be called the fundamental frequency and twice this frequency, the first harmonic.

At a somewhat smaller Reynolds number, $Re_L = 6.4 \times 10^4$, the linear region is longer in terms of wake widths and the peak in the spectrum becomes narrower than in the case of the higher Reynolds number (Fig. 6). The length of the linear region and the lower freestream fluctuations in the most unstable frequency range appears to affect the sharpness of the energy peak in the spectrum. Again a further increase of the fundamental frequency component and a further tuning about

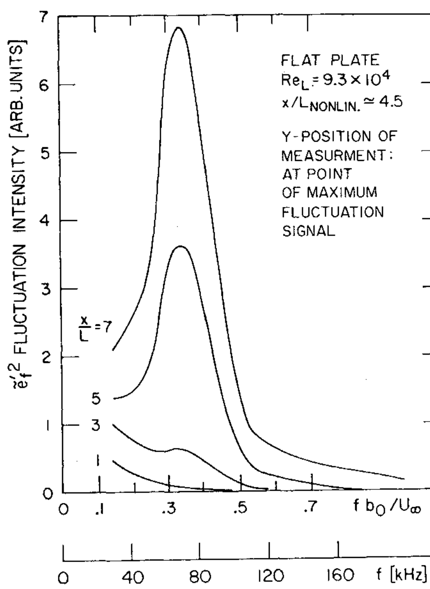

Fig. 4 Development of power spectra of fluctuations in flat-plate wake at $Re_L = 9.3 \times 10^4$.

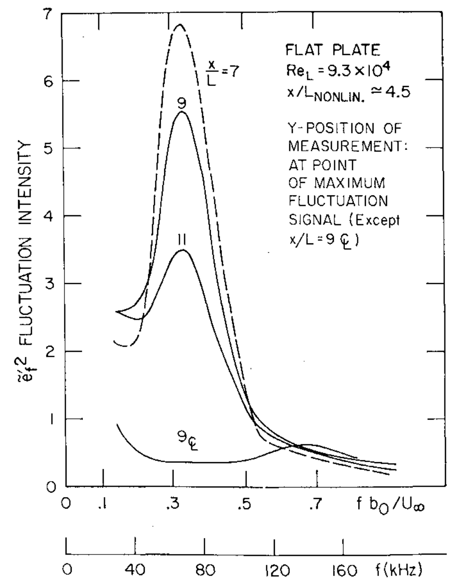

Fig. 5 Power spectra continued from Fig. 4.

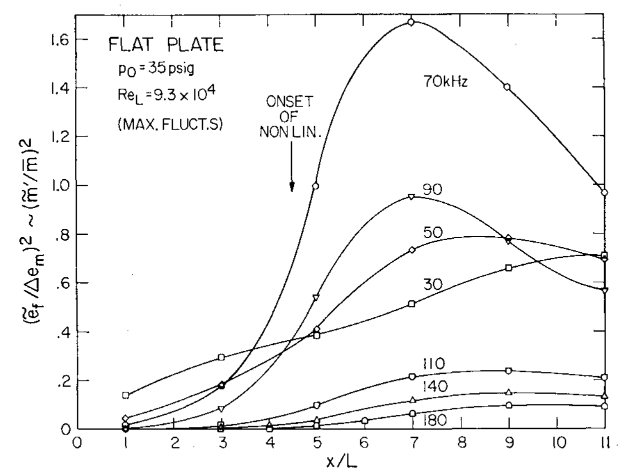

Fig. 7 Axial fluctuation development at several frequencies.

this frequency is observed at the beginning of the nonlinear region. On the wake axis a maximum is observed at twice the fundamental frequency (the first harmonic). Very similar observations were made in wakes of 20-deg wedges at Reynolds numbers of $Re_H = 1.9 \times 10^4$ and 2.8×10^4. Therefore, these frequency distributions for the wakes of wedges are not shown.

Returning to the wake of a flat plate at $Re_L = 9.3 \times 10^4$, the axial development of the maximum fluctuations at various frequencies are compared with each other (Fig. 7). All fluctuations are still growing at and beyond the onset of nonlinearity, but as also seen from the frequency spectra, the decay of the previously most unstable frequency components (70 kHz and 90 kHz) occurs sooner and at a faster rate than the decay of the larger frequency components. At the lowest frequency (30 kHz) the fluctuations are still increasing at $x/L = 11$.

On the wake axis a most dramatic rise in fluctuation intensity occurs as the wake becomes nonlinear (Figs. 3, 8, and 9). As already noted by Batt and Kubota,[4] this rise of the fluctuation signal on the wake axis occurs somewhat ahead of the rapid increase of the wake growth. Note also that the initial growth rate of the harmonic (140 kHz) is nearly twice that of the fundamental.

A comparison of the fluctuation profiles across the wake at these two frequencies is quite interesting. The fluctuation profiles of the fundamental frequency is the typical double peak profile with no energy on the wake axis in the linear region (Fig. 3). At the first harmonic there is practically no energy in the linear region but as the breakdown of the laminar flow occurs, the signal increases rapidly and the profile changes from the double peak profile to a profile with the maximum on the wake axis (Fig. 8). The fluctuation distributions at 90 kHz are quite similar to those at 70 kHz and therefore are not shown. However, the distributions at 180 kHz change character even more so than those at 140 kHz. From a double peak profile at $x/L = 3$ the fluctuation distribution changes to a triple peak profile at $x/L = 6$ and finally to a large peak on the wake axis at $x/L = 9$ and 11 (Fig. 9).

These measurements confirm that also in these slender body wakes at Mach 6, the linear instability region (in which the mean flow is laminar) is followed by a nonlinear instability region in which the mean flow changes rapidly initially and the fluctuation spectrum undergoes dramatic changes. In none of the present experiments was it possible to make measurements far enough downstream to reach "fully developed turbulent flow."

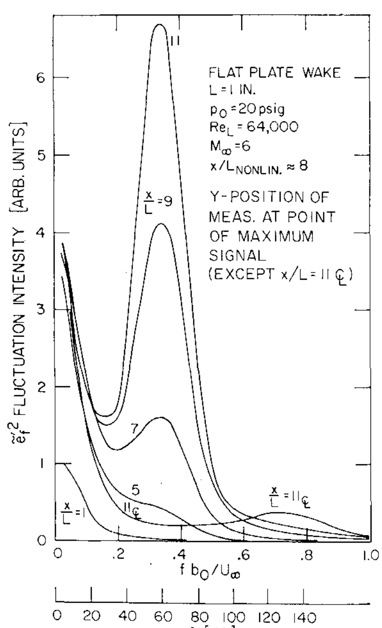

Fig. 6 Power spectra of fluctuations in flat-plate wake at $Re_L = 6.4 \times 10^4$.

Discussion of Results

Linear Instability Region

Inviscid Linear Stability Calculations

Wake profiles are dynamically unstable because at some point in the profile an extremum in the density-vorticity product (Lees and Lin[5]) occurs. Mack[22] developed a numerical method to determine the eigenvalues and eigenfunctions of the compressible inviscid (and viscous) stability equations for a given boundary-layer profile. Phase velocity, rate of amplification, wave number and fluctuation profiles across the wake may be calculated. The computer program of Mack for the inviscid compressible stability equations was modified for wake profiles.[12] The boundary conditions were chosen to correspond to the antisymmetric mode of fluctuation which has been shown to be the most unstable mode (Lees and Gold[5]). Earlier experimental work in wakes of cylinders[10] and a comparison with theoretical calculations[12] indicated that the amplification rate depends significantly on the wake profile shape. Therefore, measured mean velocity and temperature profiles were used to calculate the stability of these flows.

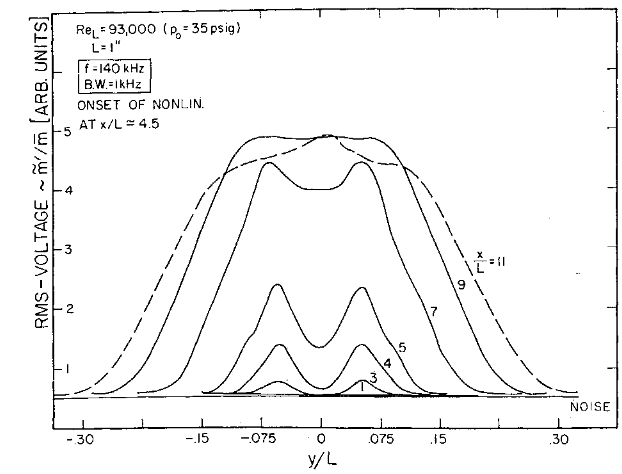

Fig. 8 Fluctuation amplitude distributions across wake at 140 kHz.

Amplification of Fluctuations

The experimental and theoretical rates of amplification were compared for the wake of a flat plate two plate lengths behind the plate for which the development of the frequency spectrum is shown in Figs. 4 and 5. The maximum experimental amplification rate and the most unstable frequency agree well with the theoretical values (Fig. 10). However, for $\alpha b = 1.24$, linear theory predicts a nearly neutral oscillation but experimentally, an amplification was observed. Since the fluctuations are small and quite hard to measure accurately at this high frequency, more evidence is required to conclude whether or not this discrepancy between experiment and theory is significant.

Mass Flow Fluctuation Amplitude Distribution

As previously mentioned, at high enough overheats, the hot-wire voltage fluctuation is essentially proportional to the mass flow fluctuations. The distribution of mass-flow fluctuations across the wake as calculated from stability theory at a certain wave number should be comparable to the hot-wire fluctuation trace at the corresponding frequency. This comparison is shown for one case in Fig. 11. The agreement is quite good which indicates the validity of linear stability theory, assuming that the hot-wire measures the mass flow distribution.

It is worth noting that the critical point (= singular point, where $u = c$ for a neutrally stable wave and where the density-vorticity product has an extremum) is near the peak in the hot-wire trace (Fig. 11).

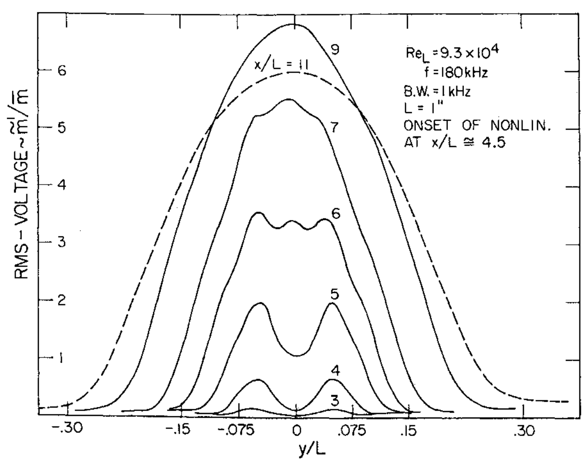

Fig. 9 Fluctuation amplitude distributions across wake at 180 kHz.

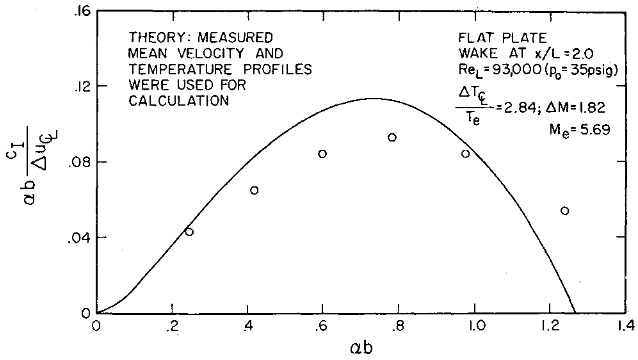

Fig. 10 Rates of amplification of fluctuations in linear instability region.

Frequency Spectrum Development-Comparison between Experiment and Linear Theory

Since linear stability theory appears to apply in the linear region, one may calculate the axial development of the frequency distribution of fluctuations for a given initial frequency distribution. This was done for the wake of a wedge at $Re_H = 28{,}000$ in which case the linear region extends to $x/H \approx 21$. Given the mean wake flow (taken from Batt and Kubota's measurements)[15] and the normalized rate of amplification as function of wave number (corresponding to a frequency) the growth rate of hot-wire fluctuations, $d\ \ln\tilde{e}_f'/d(x/H)$ may be calculated from Eq. (3) as function of (x/H). The theoretical amplification rate used for this purpose, shown in Fig. 10, is the one calculated for a flat plate wake profile, ignoring the slight differences in profile shapes. The rate of amplification $\alpha c_I H/c_g \sim d\ \ln\tilde{e}_f'/d(x/H)$ vs x/H may be numerically or graphically integrated to obtain the hot-wire fluctuation as function of frequency, x-station and initial magnitude. The results are shown in Fig. 12. As the flow moves from $x/H = 5$ to $x/H = 15$ the measured fluctuations grow nearly at the same rate as the ones calculated from linear theory, except at low frequencies (below 20 kHz). In both experiment and theory a distinct peak occurs at 45 and 52 kHz, respectively. Had the initial fluctuation spectrum at $x/H = 5$ been a uniform spectrum, the peak at $x/H = 15$ would have occurred at $f = 65$ kHz.

These results show that linear stability theory predicts quite well the actual development of the power spectrum. Out of an initially monotonically decreasing spectrum a definite peak in a preferred frequency range develops, corresponding closely to the measurement.

However, at low frequencies, the experimentally measured instability is not predicted by linear theory. It should be noted, that at $f = 5$ kHz, the fluctuation signal grows rapidly between $x/H = 5$ and 10 but changes little between $x/H = 10$ and 15. These low-frequency phenomena are not understood.

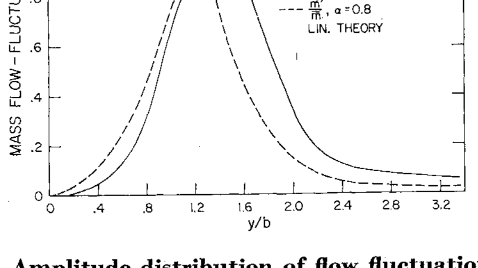

Fig. 11 Amplitude distribution of flow fluctuations (experiments and linear stability theory).

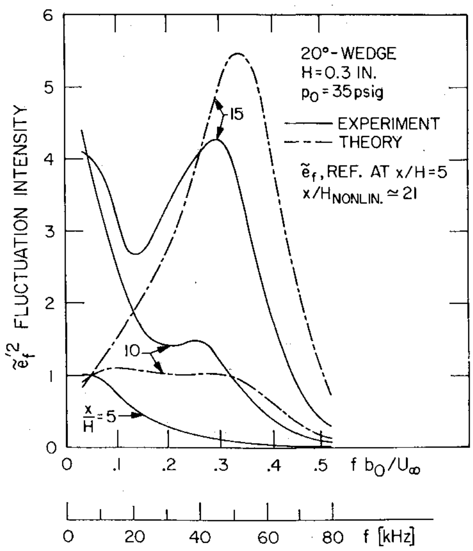

Fig. 12 Power spectra, measured in wake of wedge and calculated from linear stability theory.

Nonlinear Instability Region

Comparison with Low-Speed Wakes

Presently, the understanding of this nonlinear instability region is limited. However, the observations made in the nonlinear region bear some resemblance to the results obtained by Roshko[23] in the low-speed wake behind a circular cylinder in the irregular range ($300 < Re_d < 10^4$) and also to the nonlinear region of the low-speed wake behind a flat plate investigated by Sato and Kuriki.[8] (Sato and Kuriki conclude that "besides the difference in mechanism of generation, the double row of vortices in the wake of a cylinder might correspond to that in the nonlinear region of the wake of a flat plate.") The amplitude distributions of fluctuations across the wake at the fundamental and the first harmonic as shown in Figs. 3 and 8 at $x/L = 11$ are qualitatively similar to those shown in Fig. 11 of Roshko's paper.[23]

Frequency spectra are not given in Sato and Kuriki's paper. Therefore, a comparison is made with Roshko's frequency spectra (Fig. 16, Ref. 23). These spectra have some resemblance to those spectra of Fig. 6 at $x/L = 11$. In both cases, off the axis at the position where the fundamental frequency has its maximum amplitude, there are peaks in the spectrum at this frequency. On the wake axis, there are peaks at the first harmonic frequency.

The main difference between these fluctuation spectra is the fact that in the hypersonic wake the peaks are of finite band width whereas in the case of Roshko's cylinder wake the peaks are discrete energy peaks; they are δ-functions in the spectra which represent monochromatic waves. Judging by this comparison, one would not expect the nonlinear region of the hypersonic wakes under discussion here to be a double row of discrete vortices, but rather a nonlinear vortical motion with most of the energy in some preferred frequency range.

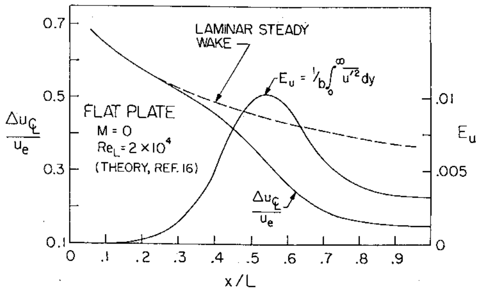

Fig. 13 Axial development of wake velocity defect and fluctuation intensity in low-speed flat-plate wake (Ref. 16).

Table 1 High-speed wakes (present experiments) $M_\infty = 6$

Model	Re_L or Re_H	$f\theta/u_\infty$	fb_0/u_∞
Flat plate	4.6×10^4	0.0122	0.312
$L = 1''$	6.4×10^4	0.0142	0.340
	9.3×10^4	0.0142	0.329
20°-wedge	9.6×10^3	0.0140	0.337
$H = 0.15''$	1.4×10^4	0.0140	0.344
20°-wedge	1.92×10^4	0.0145	0.327
$H = 0.30''$	2.8×10^4	0.0140	0.307

Fundamental Fluctuation Frequency

The frequency spectra shown in Figs. 4, 5, and 6 indicate that the peak in the spectrum, which corresponds to the most unstable frequency in the linear region does not change in the nonlinear region even though the mean wake width changes rapidly. Furthermore, in the initial stage of this nonlinear region the peak at the most unstable frequency becomes even narrower. The "peak" frequencies for all spectra measured in the present experiments were normalized by a characteristic length scale and the freestream velocity (which is nearly the edge velocity of the wake). The length scales used are the wake width b_0, defined by the velocity profile in the linear region just downstream of the wake neck, and the momentum thickness. The Strouhal numbers obtained for all present experiments are tabulated in Table 1. Both Strouhal numbers are nearly constant.

Strouhal numbers for incompressible wakes are shown in Table 2. The comparison with the Stouhal numbers of the incompressible flat plate wakes shows that the wake width is the length scale with which the fundamental frequency of oscillation scales. The Strouhal numbers are $S \doteq 0.3$ as compared to $S \doteq 0.32$ for the hypersonic wakes. This rather interesting result may be partly explained by linear stability theory.

In a previous paper[10] it was noted that the amplification curves of linear instabilities for Gaussian wake profiles are essentially the same for incompressible wakes and the hypersonic wakes considered here if the wave number is normalized by the physical wake thickness. Assuming the wake thickness is constant in the linear region and the profiles are Gaussian, the nondimensional most unstable wave number is constant. As shown in the preceding section on the "linear region," the initial spectrum at the beginning of the wake, the wave speed and the most unstable wave number determine the most unstable frequency. These quantities vary somewhat in the actual wake flows, but apparently vary in such a way that the Strouhal number is nearly the same for both hypersonic and low-speed slender body wakes.

In the case of blunt body wakes the situation is more complicated. At low enough Reynolds numbers, such as in case of Kovasznay's[24] wake of a cylinder at $Re_d = 56$, the vortex street also developed from a linear instability of the laminar wake which explains that the Strouhal number has nearly the same value as in case of the slender body wakes. However, it

Table 2 Low-speed wakes

Model	Investigators	$f\theta/u_\infty$	fb_0/u_∞	Comments
Flat plates $Re_L = 10^4$	Taneda[7]	0.064	0.29	Average value
Flat plates $Re_L = 1.9 \times 10^5$	Sato & Kuriki[8]	0.066	0.30	
Cylinder $Red = 56$	Kovasznay[24]	0.26		
Cylinder $300 < Red < 10^4$+	Roshko[23] (p. 17)		≈0.28	Average value
Blunt and slender bodies at angle of attack	Fage & Johansen[25]		0.27–0.33 (0.28)	Average value

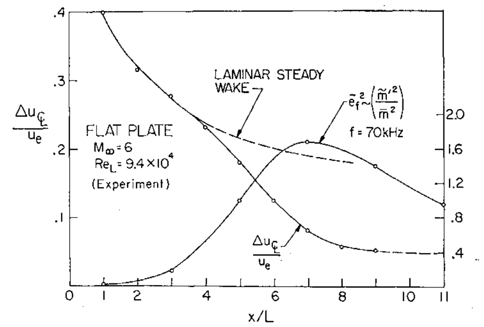

Fig. 14 Axial development of wake velocity defect and fluctuation intensity in flat-plate wake at Mach 6.

is surprising that in all the other blunt body wakes, quoted in Table 2, the Strouhal numbers are also close to those of the slender body wakes, even though the mechanism of generation of the vortex streets is quite different.

Suggestion for a Theoretical Model

It has been shown that there exist similarities in the linear and nonlinear instability mechanism in compressible and incompressible wakes. Especially, it has been shown that the energy peak in the frequency spectrum (fundamental frequency) characterized by the Strouhal number $S = fb_0/u_e \doteq 0.32$ persists also in compressible wakes in the nonlinear region. This fact suggests that the same approach successfully employed by Ko, Kubota and Lees[16] in low-speed wakes might be used as the simplest model for calculating the flow in the nonlinear region. They used an integral technique to investigate the interaction between a single frequency finite amplitude disturbance with the mean flow in a laminar incompressible wake behind a flat plate, and good agreement with Sato and Kuriki's experiments was obtained. Their results for the velocity defect on the wake axis and the development of the fluctuation intensity are shown in Fig. 13 for a $Re_L = 2 \times 10^4$. Note that the drastic changes in velocity defect and fluctuation intensity occur in one-half plate length.

Qualitatively similar results are obtained for the hypersonic wake at $Re_L = 9.3 \times 10^4$, shown in Fig. 14 where the experimentally determined velocity defect and the maximum mass-flow fluctuations are plotted. Note, however, that the region of rapidly changing flow extends over 5 plate lengths rather than 0.5 plate length as in the low-speed wake.

References

[1] Goldburg, A., "A Summary Analysis of Laboratory Hypersonic Wake Fluid Mechanics Transition Experiments," AIAA Paper 67-33, New York, 1967.

[2] Pallone, A. J. et al., "Hypersonic Laminar Wakes and Transition Studies," *AIAA Journal*, Vol. 2, No. 5, May 1964, pp. 855–863.

[3] Bailey, A. B., "Observations of Sphere Wakes Over a Wide Range of Velocities and Ambient Pressures," AEDC-TR-68-112, Sept. 1968.

[4] Lees, L., "Hypersonic Wakes and Trails," *AIAA Journal*, Vol. 2, No. 3, March 1964, pp. 417–428.

[5] Lees, L. and Gold, H., "Stability of Laminar Boundary Layers and Wakes at Hypersonic Speeds. Part I. Stability of Laminar Wakes," *Proceedings of International Symposium on Fundamental Phenomena in Hypersonic Flow*, Cornell University Press, 1966.

[6] Lees, L. and Lin, C. C., "Investigation of the Stability of a Laminar Boundary Layer in a Compressible Fluid," TN 1115, 1946, NACA.

[7] Taneda, S., "Oscillation of the Wake Behind a Flat Plate Parallel to the Flow," *Journal of the Physical Society of Japan*, Vol. 13, No. 4, April 1958, pp. 418–425.

[8] Sato, H. and Kuriki, K., "The Mechanism of Transition in the Wake of a Thin Flat Plate Placed Parallel to a Uniform Flow," *Journal of Fluid Mechanics*, Vol. 11, Pt. 3, Nov. 1961, pp. 321–352.

[9] Kendall, J. M., Jr., "Stability of a Supersonic Wake Flow, Pt. II," *JPL Space Programs Summary No. 37-34*, California Institute of Technology, Jet Propulsion Lab., Pasadena, Calif., Vol. IV, pp. 212–213.

[10] Behrens, W., "Far Wake Behind Cylinders at Hypersonic Speeds: II. Stability," *AIAA Journal*, Vol. 6, No. 2, Feb. 1968, pp. 225–232.

[11] Lewis, J. E. and Behrens, W., "Fluctuation Measurements in the Near Wake of a Wedge with and without Base Injection," *AIAA Journal*, Vol. 7, No. 4, April 1969, pp. 664–670.

[12] Kubota, T. and Behrens, W., "A Note on Linear Instability of Far Wake Flow," *AIAA Journal*, Vol. 8, No. 6, June 1970, pp. 1178–1179.

[13] Kendall, J. M., Jr., private communication; also Betchov, R. and Criminale, W. O., Jr., *Stability of Parallel Flows*, Academic Press, New York, 1967, Fig. 31.1.

[14] Demetriades, A. and Behrens, W., "Hot Wire Measurements in the Hypersonic Wakes of Slender Bodies," International Memo. No. 14, May 15, 1963, Graduate Aeronautical Labs., California Institute of Technology, Hypersonic Research Project; also Demetriades, A., "Hot Wire Measurements in the Hypersonic Wakes of Slender Bodies," *AIAA Journal*, Vol. 2, No. 2, Feb. 1964, pp. 245–250.

[15] Batt, R. G. and Kubota, T., "Experimental Investigation of Far Wakes Behind Two-Dimensional Slender Bodies at $M_\infty = 6$," *AIAA Journal*, Vol. 7, No. 11, Nov. 1969, pp. 2064–2071.

[16] Ko, D. R. S., Kubota, T., and Lees, L., "Finite Disturbance Effect on the Stability of a Laminar Incompressible Wake Behind a Flat Plate," *Journal of Fluid Mechanics*, Vol. 40, Pt. 2, 1970, pp. 315–341.

[17] Kovasznay, L. S. G., "Hot-Wire Method," *High Speed Aerodynamics and Jet Propulsion*, Vol. IX, Sect. F2, Princeton University Press, Princeton, N.J., 1954.

[18] Morkovin, M. V. and Phinney, R. E., "Extended Applications of Hot-Wire Anemometry to High-Speed Turbulent Boundary Layers," AFOSR TN-58-469, 1958, Dept. of Aeronautics, Johns Hopkins Univ.

[19] Morkovin, M. V., "Fluctuations and Hot-Wire Anemometry in Compressible Flows," AGARDograph 24, Nov. 1956.

[20] Demetriades, A., "Turbulence Measurements in an Axisymmetric Compressible Wake," *The Physics of Fluids*, Vol. 11, Sept. 1968, pp. 1841–1852.

[21] Kistler, A. L., "Fluctuation Measurements in a Supersonic Turbulent Boundary Layer," *The Physics of Fluids*, Vol. 2, 1959, pp. 290–296.

[22] Mack, L. M., "Computations of the Stability of the Laminar Compressible Boundary Layer," *Methods of Computational Physics*, Vol. 4, Academic Press, New York, 1965, pp. 247–299.

[23] Roshko, A., "On the Development of Turbulent Wakes from Vortex Streets," TR 1191, 1954, NACA.

[24] Kovasznay, L. S. G., "Hot-Wire Investigation of the Wake Behind Cylinders at Low Reynolds Numbers," *Proceedings of the Royal Society* (*London*), *Ser. A*, Vol. 198, No. 1053, Aug. 1949, pp. 174–190.

[25] Fage, A. and Johansen, F. C., "The Structure of Vortex Streets," *Philosophical Magazine*, Ser. 7, Vol. 5, No. 28, Feb. 1928, pp. 417–441.